Vietnam National University
University of Science
Faculty of Mathematics and Computer Science

MATHEMATICS COURSE CATALOG

2015 UNDERGRADUATE PROGAM

Composed by Le Chieu Hoang Nguyen, 2019

Note: Since 2016 the course numbers were changed, in particular TTH were changed to MTH, however the course descriptions were same.

Algebra A1

Subject code: TTH001
Recommended Prerequisites: _
Prerequisites: _
Student Workload: 182.5 hours, of which 37.5 hours of studying theories, 25 hours of practicing, 120 hours of independent study.
Number of Credits: 4

Contents:

Chapter 1: Matrices and Systems of linear equations
1.1. Definitions and notations
1.2. Matrix operations
1.3. Invertible matrices
1.4. Matrix equations
1.5. Systems of linear equations
1.6. Gaussian elimination

Chapter 2: Determinant
2.1. Permutations
2.2. Definition of determinant and basic properties
2.3. Determinant and invertible matrices
2.4. Cramer's rule
2.5. Determinant and rank of a matrices

Chapter 3: Vector spaces
3.1. Definition and basic properties
3.2. Linear combinations
3.3. Linear independence and linear dependence
3.4. Subspaces, generating sets, bases and dimensions
3.5. Row spaces
3.6. Solution spaces
3.7. Sum spaces
3.8 Coordinates and matrices of change of basis

Chapter 4: Linear transformations
4.1. Definition and basic properties
4.2. Kernels and images of linear transformations

4.3. Matrix representations of linear transformations
4.4. Dual spaces

References

1. *Đại số tuyến tính và ứng dụng*, Tập 1, Bùi Xuân Hải, Trần Ngọc Hội, Trịnh Thanh Đèo, Lê Văn Luyện, 2009.
2. *Giáo trình Đại số tuyến tính*, Ngô Việt Trung, 2001.
3. *Đại số tuyến tính*, Nguyễn Hữu Việt Hưng, 2004.

Higher Algebra

Subject code: TTH006
Recommended Prerequisites: _
Prerequisites: _
Student Workload: 182.5 hours, of which 37.5 hours of studying theories, 25 hours of practicing, 120 hours of independent study.
Number of Credits: 4

Contents:

Chapter 1: Groups
1.1. Binary operations
1.2. Semigroups, monoids and groups
1.3. Subgroups, Lagrange's theorem
1.4. Permutation groups and alternating groups
1.5. Cyclic groups
1.6. Order of an element in a group
1.7. Normal subgroups and factor groups
1.8. Group homomorphisms, Isomorphism theorems

Chapter 2: Rings, Integral domains and Fields
2.1. Definitions of rings, integral domains and fields
2.2. Two examples: Z and Z_n
2.3. Ideals and factor rings
2.4. Ring homomorphisms and Isomorphism theorems
2.5. Chinese remainder theorem
2.6. Relations between integral domains and fields
2.7. Characteristic of a fields
2.8. The field of quotients of an integral domain

Chapter 3: Polynomial rings
3.1. Definition and basic properties of a polynomial ring over a field
3.2. Euclidean algorithm, derivatives and multiple solutions, Taylor expansion of polynomials over a field
3.3. Irreducible polynomials over a field
3.4. Fundamental Theorem of Algebra
3.5. Polynomials over R and C
3.6. Polynomials over Q, Eisenstein's criterion

References
1. *Đại số đại cương*, Nguyễn Viết Đông, Trần Ngọc Hội, 2005.
2. *Đại số đại cương*, Hoàng Xuân Sính, 1997.
3. *Đại số đại cương*, Nguyễn Hữu Việt Hưng, 1998.
4. *Đại số đại cương*, Mỵ Vinh Quang, 1998.
5. *Bài tập Đại số đại cương*, Bùi Huy Hiền, Nguyễn Hữu Hoan, Phan Doãn Thoại, 1985.

Analysis A1 - Basic Analysis

Subject code: TTH021
Recommended Prerequisites: _
Prerequisites: _
Student Workload: 140 hours, of which 25 hours of studying theories, 25 hours of practicing, 90 hours of independent study.
Number of Credits: 3

Contents:

Chapter 1: Sets and functions
1.1. Sets
1.2. Relations in a set
1.3. Functions

Chapter 2: Real numbers
2.1. Intergers, rational numbers and real numbers
2.2. Supremum and Infimum

Chapter 3: Real sequences and Series
3.1. Sequences, Limit of a sequence and basic properties

3.2. Liminf and Limsup
3.3. Series, Limit of a series and basic properties
3.4. Mathematica for investigating sequences and series

Chapter 4: Continuous real functions
4.1. Definition and basic properties
4.2. Continuity of basic functions
4.3. Continuous functions on a closed interval
4.4. Uniform convergence

References
1. M.I. Abell and J.P. Braschon, *Mathematica by example*. Academic Press, New York, 1997.
2. S.I. Grossman, *Calculus*. Harcourt Brace College Publishers, New York, 1992.
3. Dương Minh Đức, *Giáo trình Giải tích 1*. Nhà xuất bản Thống Kê, Tp Hồ Chí Minh, 2006.
4. W. Rudin, *Principles of mathematical analysis*. McGraw-Hill, New York, 1964.
5. S. Wolfram, *Mathematica*. Cambridge University Press, 1996.

Analysis A1 – Calculus

Subject code: TTH022
Recommended Prerequisites: _
Prerequisites: _
Student Workload: 140 hours, of which 25 hours of studying theories, 25 hours of practicing, 90 hours of independent study.
Number of Credits: 3

Contents:

Chapter 1: Differentiation
1.1. Limit of a function and basic properties
1.2. Derivatives and basic properties
1.3. Derivatives of basic functions
1.4. Properties of differentiable functions on an open interval
1.5. Higher order derivatives
1.6. Taylor expansions
1.7. Approximation methods

1.8. Applications of derivatives: investigation of functions
1.9. Mathematica for differentiation

Chapter 2: Integration, Riemann integrals
2.1. Definition and basic properties of Riemann integrals
2.2. Basic theorems of integration
2.3. Improper integrals
2.4. Methods of integration
2.5. Mathematica for integration

References
1. M.I. Abell and J.P. Braschon, *Mathematica by example*. Academic Press, New York, 1997.
2. S.I. Grossman, *Calculus*. Harcourt Brace College Publishers, New York, 1992.
3. Dương Minh Đức, *Giáo trình Giải tích 1*. Nhà xuất bản Thống Kê, Tp Hồ Chí Minh, 2006.
4. W. Rudin, *Principles of mathematical analysis*. McGraw-Hill, New York, 1964.
5. S. Wolfram, *Mathematica*, Cambridge University Press, 1996.

Analysis A2

Subject code: TTH023
Recommended Prerequisites: _
Prerequisites: _
Student Workload: 225 hours, of which 50 hours of studying theories, 25 hours of practicing, 150 hours of independent study.
Number of Credits: 5

Contents: An elementary introduction of metric spaces, normed spaces and R^n. Properties of continuous functions and of differentiable functions on R^n. Theory of real series, power series and series of functions.

Chapter 1: Metric spaces, normed spaces and R^n
1.1. Definitions of metric and norm
1.2. Open and closed subsets of a metric space
1.3. The space R^n
1.4. Paths in R^n
1.5. Surfaces in R^n

Chapter 2: Continuous functions
2.1. Continuous functions on a metric space
2.2. Uniform convergence on a compact set
2.3. Continuous real functions defined on a metric space or R^n
2.4. Properties of continuous functions on a compact set

Chapter 3: Differentiation of multivariable functions
3.1. Partial derivatives
3.2. Differentiability, derivatives of composite functions
3.3. Implicit functions and inverse functions
3.4. Geometric meanings
3.5. Higher order partial derivatives and Taylor formular
3.6. Mathematica for differentiation

Chapter 4: Extreme values of multivariable functions:
4.1. Local extreme values
4.2. Conditional extreme values
4.3. Maxima and minima

Chapter 5: Number series
5.1. Definition
5.2. Positive series and the comparision test
5.3. Cauchy and D'Alembert criterions
5.4. Integral criterion
5.5. Absolutely convergent series
5.6. Semi-convergent series
5.7. Mathematica for investigating series

Chapter 6: Series in a normed spaces
6.1. Convergence of sequences, sequences in $C[a,b]$
6.2. Properties of convergent sequences
6.3. Definitions of series and convergence
6.4. Convergence in $C[a,b]$
6.5. Derivatives and integrals of series in $C[a,b]$
6.6. Cauchy sequences and the completeness of $C[a,b]$

Chapter 7: Power series
7.1. Definition, radius of convergence
7.2. Power series of basic functions, Taylor series
7.3. Operations on power series

7.4. Expansions of power series
7.5. Derivatives and integrals of power series
7.6. Applications of power series

References

1. Đặng Đình Áng, Chu Đức Khánh, Đinh Ngọc Thanh, *Vi phân Hàm nhiều biến*. ĐHKH Tự Nhiên, 2000.
2. J. Cooper, *A Matlab - Companion for multivariable calculus*. Academic Press, 2001.
3. B. Hunt, R. Lipsman and J. Rosenberg, *A guide to Matlab for beginner's and experienced users*. Cambridge University Press, 2001.
4. Wilfred Kaplan, *Advanced Calculus*. Addison-Wesley, 1993.
5. S. Lang, *A second course in calculus*. Addison-Wesley, Reading, 1969.
6. Nguyễn Đình Phư, Nguyễn Công Tâm, Đinh Ngọc Thanh and Đặng Đức Trọng, *Giáo trình giải tích hàm một biến*. NXB ĐHQG, Thành phố Hồ Chí Minh, 2002.
7. Nguyễn Đình Phư, Nguyễn Công Tâm, Đinh Ngọc Thanh and Đặng Đức Trọng. *Giáo trình giải tích hàm nhiều biến*, NXB ĐHQG,Thành phố Hồ Chí Minh, 2002.
8. S.M. Nikolsky, *A course of mathematical analysis*. Mir, Moscow, 1981.

Analysis A3

Subject code: TTH024
Recommended Prerequisites: Analysis A1, Analysis A2
Prerequisites: _
Student Workload: 182.5 hours, of which 37.5 hours of studying theories, 25 hours of practicing, 120 hours of independent study.
Number of Credits: 4

Content: Multiple integrals, line integrals, Green theorem, surface integrals, Stokes theorem.

Chapter 1: Multiple integrals
1.1. Riemann integrals over a box: coherent theory, precise definition via upper and lower sums (Darboux).
1.2. Integrability condition: show that if a function is continuous then it is integrable, measure zero, volume zero, state the necessary-and-sufficient condition for integrability.
1.3. Integrals over an arbitrary subset of R^n: volume, condition for having volume, properties of integral.
1.4. Fubini theorem (with proof)

1.5. Change of variables: review of multivariable differentiation, change of variables formula (not proven but explained).
1.6. Applications: quantitative meaning and geometric meaning of integrals, applications in physics and probability (evaluate $\int \exp\left(-x^2\right)$).

Chapter 2: Line integrals
2.1. Line integrals of the first and second kinds: meaning, dependence on parametrization (stated rigorously, not proven).
2.2. Newton - Leibniz theorem, preservative fields (physical meaning)
2.3. Green theorem (proven stricly)

Chapter 3: Surface integrals
3.1. Surface integrals of the first and second kinds: meaning, dependence on parametrization (stated rigorously, not proven).
3.2. Stokes theorem, Gauss - Ostrogradsky theorem (proven strictly), meaning of div and Curl (Rot),
3.3. Applications: Maxwell equations,
3.4. An introduction to differential forms (if have enough time)

Textbooks:
1. James Stewart, *Calculus: Early Transcendentals*, 7th edition. Brooks/Cole, 2012.
2. Huỳnh Quang Vũ, *Bài giảng môn Giải tích A3*,
http://www.math.hcmus.edu.vn/~hqvu/gt3.pdf

References:
1. Đặng Đình Áng, *Lý thuyết tích phân*. NXB Giáo dục, Tp. HCM,1997.
2. J. Cooper, *A Matlab companion to multivariable calculus*. Hartcourt, 2001.
3. S. Lang, *Undergraduate Analysis*. Springer, 1997.
4. Nguyễn Đình Phư, Nguyễn Công Tâm, Đinh Ngọc Thanh and Đặng Đức Trọng, *Giáo trình giải tích hàm nhiều biến*. NXB ĐHQG,Thành phố Hồ Chí Minh, 2002.
5. W. Rudin, *Principles of mathematical analysis*. Mc Graw–Hill, 1976.
6. M. Spivak, *Calculus on manifolds*. Addison-Wesley, 1965.
7. Nguyễn Đình Trí, Tạ Văn Đĩnh, Nguyễn Hồ Quỳnh, *Toán học cao cấp, tập 3, Phép tính giải tích nhiều biến số*. NXB Giáo dục 2007.
8. Jerrold E. Marsden and Anthony J. Tromba, *Vector calculus*. Freeman, 2003.

Softwares: Matlab, Sage, Maxima, Maple, Mathematica.

Analysis A4

Subject code: TTH025
Recommended Prerequisites: Analysis A1, Analysis A2
Prerequisites: _
Student Workload: 127.5 hours, of which 37.5 hours of studying theories, 90 hours of independent study.
Number of Credits: 3

Contents: First order differential equations; Existence - uniqueness theorem for solution of Cauchy problem; Second and higher- order linear differential equations, Introduction to systems of first oder differential equations.

Chapter 1: First order differential equations
1.1. Solutions of first order linear differential equations. Existence and uniqueness of solution of Cauchy problems.
1.2. Separation of variables method for finding solutions
1.3. Types of solutions of first order differential equations
1.4. Method for finding solutions of homogeneous differential equations
1.5. Method for finding solutions of total differential equations
1.6. Method for finding solutions of differential equations of form $y'=f(ax+by)$
1.7. Method for finding solutions of Bernoulli and Riccati equations
1.8. Existence of solutions of differential equations of form $y'=f(x,y)$

Chapter 2: Higher order differential equations
2.1. Existence and uniqueness of solutions of higher order differential equations
2.2. System of fundamental solutions of a homogeneous linear differential equation of degree n
2.3. Wronskian determinant of solutions; Ostrogradski-Liouville formulae
2.4. Method for finding special solutions
2.5. Solutions of non-homogeneous linear differential equations of degree n
2.6. System of fundamental solutions of a homogeneous Euler equation
2.7. Solutions of non-homogeneous Euler equations

Chapter 3: Introduction to Systems of first order linear differential equations
3.1. Linear differential operators of systems of 1^{st} order linear differential equations
3.2. Relationship between higher order differential equations and 1st order differential equations
3.3. Existence and uniqueness of solutions of Cauchy problems for systems of 1st order linear differential equations
3.4. System of fundamental solutions of a systems of 1st order linear differential equations

References

1. Nguyễn Thanh Vũ, *Phương trình vi phân*. NXB. ĐHQG Tp. HCM, 2001.
2. Edwards, Penny, *Differential equations*. Pearson Education, Inc., 2004.
3. Bruce P. Conrac, *Differential equations*. Pearson Education, Inc., 2003.
4. R. Kent Nagle, Edward B. Saff, *Fundamentals of differential equations and boundary value problems*. Addison-Wesley Publishing Company, 1993.

Practical Laboratory

Subject code: TTH091
Recommended Prerequisites: _
Prerequisites: _
Student Workload: 110 hours, of which 50 hours of practicing, 60 hours of independent study.
Number of Credits: 2

Contents:

Chapter 1: Basic Matlab
1.1. Introduction, installation
1.2. Operations, variables, vectors, matrices
1.3. Logic, graphs
1.4. Loops: for/while, if, .m file

Chapter 2: Applications to Linear algebra
2.1. Basic matrix operations
2.2. Invertible matrices, matrix equations, systems of linear equations
2.3. Determinant, solving systems of linear equations by using determinants
2.4. Characteristic polynomials, eigenvalues, eigenvectors

Chapter 3: Applications to Single variable calculus
3.1. Set operations
3.2. Problems about number sequences and series
3.3. Problems about continuous functions, limits
3.4. Problems about differentiation of single variable functions
3.5. Problems about integration of single variale functions

Chapter 4: Applications to Mechanics and Theory of Probability and Statistics
4.1. Problems about mechanics, thermaldynamics

4.2. problems about random variables
4.3. Data statistics

References

1. Brian D. Hahn, Daniel T. Valentine, *Essential MATLAB for Engineers and Scientists*. 2007.
2. J. Stewart, *Calculus: concepts and contexts*. 2002.

Measure theory and Probability

Subject code: TTH101
Recommended Prerequisites: Analysis A1, Analysis A2
Prerequisites: _
Student Workload: 182.5 hours, of which 37.5 hours of studying theories, 25 hours of practicing, 120 hours of independent study.
Number of Credits: 4

Contents: Giving the introduction of elementary probability theory, abstract measure theory, random variables, law of large numbers and limit theorems.

Chapter 1: Measure spaces, Probability spaces
1.1. Sample spaces, events
1.2. Probability of an event
1.3. Measure spaces
1.4. Probability spaces

Chapter 2: Lebesgue integrals
2.1. Measurable functions
2.2. Integrals of positive functions
2.3. Integrals of general functions
2.4. Sets of measure zero
2.5. Lebesgue measure

Chapter 3: Random variables
3.1. Definition and basic properties
3.2. Usual probability density functions
3.3. Characteristics parameters of random variables
3.4. Probability density of functions of random variables
3.5. Random vectors

Chapter 4: Limit theorems
4.1. Convergence in probability and the Weak law of large numbers
4.2. The Strong law of large numbers
4.3. The Central limit theorem
4.4. Other probability inequalities

References
1. Đ. Đ. Áng, *Nhập môn giải tích*. NXBGD, 1997.
2. Đ. Đ. Áng, *Lý thuyết tích phân*. NXBGD, 1997.
3. P. Billingsley, *Probability and measure*, 3rd edition. John Wiley & Sons, 1995.
4. W. M. Bolstad, *Introduction to Bayesian statistics*, 2nd edition. Wiley, 2007.
5. M. Bouyssel, *Intégrale de Lebesgue*. Cépaduès-éditions, 1997.
6. K. L. Chung, *A course in probability theory*, 3rd edition. Academic Press, 2001.
7. D. M. Đức, *Lý thuyết độ đo và tích phân*. NXB Đại Học Quốc Gia Tp. HCM, 2006.
8. W. Feller, *An introduction to probability theory and its applications*, Vol. I. John Wiley & Sons, 1957.
9. A. M. Mathai, H. J. Houbold, *Special functions for applied scientists*. Springer, New York, 2008.
10. P.H. Quân, Đ.N. Thanh, *Xác suất thống kê*. NXBGD, 2011.
11. S. Ross, *A first course in probability*, 5 th edition. Prentice Hall, 1998.
12. W. Rudin, *Real and complex analysis*, 3 rd edition. McGraw-Hill, 1986.
13. N. D. Tiến, V. V. Yên, *Lý thuyết xác suất*. NXBGD, 2006.
14. Y. Viniotis, *Probability and random processes for electrical engineers*. McGraw-Hill, 1998.
15. T. A. Dũng, *Lý thuyết xác suất và thống kê toán*. NXB Đại Học Quốc Gia Tp. HCM, 2007.

Linear Algebra A2

Subject code: TTH102
Recommended Prerequisites: _
Prerequisites: Algebra A1
Student Workload: 182.5 hours, of which 37.5 hours of studying theories, 25 hours of practicing, 120 hours of independent study.
Number of Credits: 4

Contents:

Chapter 1: Diagonalization
1.1. Eigenvalues and eigenvectors
1.2. Eigenspaces
1.3. Diagonalization of linear operators
1.4. Diagonalizing matrices
1.5. Applications of diagonalization

Chapter 2: Jordan normal forms
2.1. Triangulation of matrices
2.2. Hamilton - Cayley theorem
2.3. Minimal polynomials
2.4. Block triangular forms
2.5. Jordan normal forms

Chapter 3: Euclidean spaces
3.1. Dot (inner) products and Euclidean spaces
3.2. Isomorphisms between Euclidean spaces
3.3. Norms, angle between two vectors
3.4. Orthogonality, orthogonal bases and orthonormal bases
3.5. Gram-Schmidt orthogonalization
3.6. Distances, distance between a vector and a subspace
3.7. Linear operators in Euclidean spaces
3.8. Orthogonal operators
3.9. Symmetric operators

Chapter 4: Bilinear forms and quadratic forms
4.1. Matrix representations of bilinear forms, change of basis
4.2. Quadratic forms
4.3. Matrix representations of quadratic forms, change of basis
4.4. Canonical forms of quadratic forms, Lagrange's method
4.5. Orthogonal canonical forms of real quadratic forms
4.6. Real quadratic forms: normal forms, law of inertia, definite forms and Sylvester's criterion

References
1. *Đại số tuyến tính và ứng dụng*, Tập 2, Bùi Xuân Hải, Trần Ngọc Hội, Trịnh Thanh Đèo, Lê Văn Luyện, 2009.
2. *Giáo trình Đại số tuyến tính*, Ngô Việt Trung, 2001.
3. *Đại số tuyến tính*, Nguyễn Hữu Việt Hưng, 2004.

Functional Analysis

Subject code: TTH104
Recommended Prerequisites: Analysis A1, Analysis A2, Analysis A3
Prerequisites: _
Student Workload: 182.5 hours, of which 37.5 hours of studying theories, 25 hours of practicing, 120 hours of independent study.
Number of Credits: 4

Contents: Metric spaces, Normed spaces, Linear maps and fundamental theorems, Hilbert spaces.

Chapter 1: Metric spaces
1.1. Metric spaces, topology on a metric space
1.2. Convergence
1.3. Continuous maps
1.4. The contraction mapping theorem

Chapter 2: Normed spaces
2.1. Definition
2.2. Sequences and series in normed spaces
2.3. Banach spaces
2.4. The space $C[a,b]$ and its completeness
2.5. Spaces l^p and L^p

Chapter 3: Continuous linear maps on normed spaces
3.1. Continuity of linear maps
3.2. Norms of linear maps
3.3. Spaces of continuous linear maps
3.4. Hahn-Banach theorem and an introduction to some other important theorems (optional) (we do not use the notions of weak topology and topological vector spaces)

Chapter 4: Hilbert spaces
4.1. Inner products, inequalities of Schwartz and Minkowski
4.2. Hilbert spaces
4.3. Orthonormal collections
4.5. Riesz representation theorem
4.6. An appication: Fourier expansion

Chapter 5: Spectrum of compact operators (optional)

References
1. A. N. Kolmogorov, S. V. Fomin, *Introductory Real Analysis*. Dover, 1975.
2. S. Lang, *Undergraduate analysis*, 2nd ed. Springer, 1997.
3. W. Rudin, *Principles of mathematical analysis*. McGraw-Hill, New York, 1976.
4. W. Rudin, *Real and complex analysis*, 3rd edition, McGraw-Hill, New York, 1986.
5. Hoàng Tụy, *Hàm thực và Giải tích hàm*. NXB Đại Học Quốc Gia Hà Nội, 2005.
6. Erwin Kreyszig, *Introductory functional analysis and applications*. John Wiley and sons, 1978.

Discrete Mathematics

Subject code: TTH105
Recommended Prerequisites: _
Prerequisites: _
Student Workload: 182.5 hours, of which 37.5 hours of studying theories, 25 hours of practicing, 120 hours of independent study.
Number of Credits: 4

Contents:

Chapter 1: Logic
1.1. Logic propositions
1.2. Logical operators
1.3. Compound propositions
1.4. Laws of logic
1.5. Quantifiers
1.6. Rules of inference
1.7. Mathematical induction

Chapter 2: Sets and functions
2.1. Sets
2.2. Set operations
2.3. Cartesian product of sets
2.4. Functions, image and inverse image of a set through a function
2.5. Types of functions

Chapter 3: Counting
3.1. Basic rules of counting

3.2. Combinatorial analysis without repetition
3.3. Combinatorial analysis with repetition

Chapter 4: Recurrence relations
4.1. Definitions
4.2. Recurrence relations of first order
4.3. Recurrence relations of second order

Chapter 5: The set of integers
5.1. Divisibility
5.2. Greatest common divisors
5.3. Least common multiple
5.4. Relative primality
5.5. Prime numbers, prime factorizations

Chapter 6: Relations in a set
6.1. Binary relations
6.2. Properties of binary relations
6.3. Equivalence relations
6.4. Modular relations on Z_n

Chapter 7: Boolean functions
7.1. Boolean functions
7.2. Representing Boolean functions
7.3. Karnaugh maps of Boolean functions
7.4. Minimal expansions of Boolean functions
7.5. Circuit algebra

References
1. Lê Văn Hợp, *Bài giảng Toán rời rạc*.
2. Nguyễn Hữu Anh, *Toán rời rạc*. 1999.
3. Kenneth H. Rosen, *Discrete Mathematics and its application*. 2012.
4. Richard Johnsonbaugh, *Discrete Mathematics*. 2005.
5. Jacques Vélu, *Méthodes Mathématiques pour l'informatiques*. 2005.

Mathematical Statistics

Subject code: TTH107
Recommended Prerequisites: _
Prerequisites: _
Student Workload: 140 hours, of which 25 hours of studying theories, 25 hours of practicing, 90 hours of independent study.
Number of Credits: 3

Contents:

Chapter 1: Sampling
1.1. Samples, characteristics of samples
1.2. Random Sampling
1.3. Probability Functions, Distribution Functions
1.4. Sampling Distribution of Mean, Variance, Proportion

Chapter 2: Estimation
2.1. Definition: Point Estimation
2.2. Estimation criterions
2.3. Estimation methods
2.4. Interval Estimation; Confidence Interval
2.5. Coverage of confidence interval; Confidence Sets

Chapter 3: Hypothesis Testing
3.1. Definitions; Decision Rules; Decision Errors: Type I Error & Type II Error
3.2. Hypothesis Testing for One Population
3.3. P-Value
3.4. Strength of testing
3.5. Hypothesis Testing for Two Populations
3.6. Distribution Hypothesis Test
3.7. Independence Hypothesis Test

Chapter 4: Regression and Correlation
4.1. Definition; Simple Linear Regression
4.2. Equations of regression lines
4.3. Properties of Least Square Estimators
4.4. Hypothesis Testing in Simple Linear Regression

Real Analysis

Subject code: TTH300
Recommended Prerequisites: Analysis A1, Analysis A2, Analysis A3, Functional Analysis
Prerequisites: _
Student Workload: 170 hours, of which 50 hours of studying theories, 120 hours of independent study.
Number of Credits: 4

Contents: We construct spaces of functions L^p, $W^{m,p}$ and study Fourier's transformations.

Chapter 1: L^p spaces
1.1. Review the notion of Lebesgue integrals
1.2. Definition and basic properties of L^p
1.3. Reflexivity, separability and dual of L^p
1.4. Convolutions
1.5. Criterion for compactness in L^p
1.6. Fourier transforms and Plancherel transforms

Chapter 2: Sobolev spaces in one dimension
2.1. Definition of weak derivatives
2.2. Spaces $W^{1,p}(I)$ and $W_0^{1,p}(I)$
2.3. Application to the boundary value problems in dimension one

Chapter 3: Sobolev spaces in n dimensions
3.1. Definition of $W^{1,p}(\Omega)$
3.2. Extension operators and Sobolev inequalities, compact embedding theorems
3.3. Trace and $W_0^{1,p}(\Omega)$ spaces
3.4. Application to the elliptic boundary value problems

References
1. Haim Brezis, *Functional Analysis, Sobolev Spaces and Partial Differential Equations*. Springer, 2011.

Nonlinear Functional Analysis

Subject code: TTH301
Recommended Prerequisites: Analysis A1, Analysis A2, Analysis A3, Functional Analysis

Prerequisites: _
Student Workload: 170 hours, of which 50 hours of studying theories, 120 hours of independent study.
Number of Credits: 4

Contents:

Chapter 1: The contraction mapping theorem
1.1. Theorem
1.2. Existence and approximation of solutions to Fredholm and Volterra equations

Chapter 2: Compactness method
2.1. Topological degrees of compact vector fields
2.2. Leray-Schauder fixed point theorem and its applications to integral equations

Chapter 3: Differentiation in normed spaces
3.1. Differentiability
3.2. Inverse function theorem and applications

References
1. J. Dieudonné, *Foundations of modern analysis*. Academic Press, New York, 1960.
2. D. M. Đức, *Giải tích hàm*. NXB ĐHQG Tp Hồ Chí Minh, 2000.
3. J. T. Schwartz, *Nonlinear Functional analysis and its applications*, Vol.I. Springer, New York, 1988.

Complex Variable Functions

Subject code: TTH304
Recommended Prerequisites: Analysis A1, Analysis A2, Analysis A3
Prerequisites: _
Student Workload: 170 hours, of which 50 hours of studying theories, 120 hours of independent study.
Number of Credits: 4

Contents: Basic properties of complex numbers and complex functions. Analytic functions, the power expansion, line integral and the theory of residus.

Chapter 1: Complex numbers
1.1. Complex numbers and algebraic operations
1.2. The complex plane
1.3. Rational exponents
1.4. Riemann sphere

Chapter 2: Differentiability
2.1. Limits and continuity
2.2. Differentiability
2.3. Analyticity
2.4. Harmonic functions

Chapter 3: Elementary complex functions
3.1. Power functions
3.2. Trigonometric functions and hyperbolic functions
3.3. Logarithm functions
3.4. Trigonometric functions and inverse hyperbolic functions
3.5. Ramification

Chapter 4: Integrals on the complex plane
4.1. Line integrals
4.2. Green's theorem
4.3. Path independence and primitives
4.4. Cauchy integral formulae

Chapter 5: Power series
5.1. Convergence of series of complex numbers
5.2. Uniform convergence of series of complex functions
5.3. Power series and Taylor series
5.4. Taylor series expansion technique
5.5. Laurent series

Chapter 6: Residuation and applications
6.1. Singular points
6.2. Definition of residues
6.3. Calculate residues
6.4. Application of residue calculus to evaluate integrals
6.5. Application to calculate Fourier and Laplace transforms

Textbooks
1. Theodore Gamelin, *Complex Analysis*. Springer, 2001.
2. A. David Wunsch, *Complex variables with applications*. Pearson-Addison Wesley, 2005.

References
1. L. V. Ahlfors, *Complex Analysis*. McGraw-Hill, New York, 1966.
2. Henri Cartan, *Théorie élémentaire des fonctions analytiques d'une ou plusieurs variables complexes*. Hermann, Paris, 1961.

Equations of Mathematical Physics

Subject code: TTH305
Recommended Prerequisites: Analysis A1, Analysis A2, Analysis A3, Analysis A4
Prerequisites: _
Student Workload: 170 hours, of which 50 hours of studying theories, 120 hours of independent study.
Number of Credits: 4

Contents: Linear ordinary equations of order 2; Wave equations; Heat equations; Laplace equations.

Chapter 1: Second order linear differential equations with constant coefficients
1.1. Oscillation problems
1.2. Uniqueness of solution of Cauchy problems
1.3. Solve 2^{nd} order linear differential equations with constant coefficients
1.4. Second order linear differential equations (with constant coefficients) relating to oscillation problems

Chapter 2: Equations of mathematical physics
2.1. Introduction and examples
2.2. Linear partial differential equations, properties of solutions
2.3. Classification of 2^{nd} order linear partial differential equations of two independent variables
2.4. Establishment of basic problems for 2^{nd} order linear partial differential equations

Chapter 3: Wave equations
3.1. Establishment of wave equations

3.2. Solutions of Cauchy (i.e. initial value) problem for the 1-dimensional wave equation for an infinite string
3.3. Well-posed problems; Hadamard's examples of ill posed problems
3.4. Free oscillation of a string fixed at both ends; Fourier method of separation of variables
3.5. Forced oscillation of a string fixed at both ends;
3.6. Forced oscillation of a string free at both ends;
3.7. Forced oscillation of a string pulled at both ends;
3.8. General diagram of Fourier method of separation of variables
3.9. Uniqueness of solution of mixed type problems
3.10. Oscillation of a circular membrane
3.11. Application of Laplace transforms to solve problems of mixed type

Chapter 4: Heat equations
4.1. Establishment of heat equations
4.2. Solutions of Cauchy (i.e. initial value) problem for heat equations
4.3. Heat conduction in a finite bar; Fourier method of separation of variables
4.4. Heat conduction in a finite bar with boundary conditions: non- homogeneous Dirichlet, non-homogeneous Robin, non-homogeneous Robin-Dirichlet

Chapter 5: Laplace equations
5.1. Esteblishment of boundary problems
5.2. Fundamental solutions of Laplace equations
5.3. Green's formulae
5.4. Properties of harmonic functions
5.5. Dirichlet problem for Laplace equations on a circular domain; Fourier method of separation of variables
5.6. Poisson integrals
5.7. Dirichlet problem for Laplace equations on a rectangular domain; Fourier method of separation of variables

Textbooks
Nguyễn Thành Long, *Bài giảng phương trình toán lý*. 2010.

References
1. Nguyễn Công Tâm, *Phương trình Vật lý – Toán nâng cao*. NXB ĐHQG TpHCM, 2002.
2. David Colton, *Partial Differential Equations, An introduction*. New York, Random House, 1988.
3. L. C. Evans, *Partial Differential Equations*. A.M.S, 1998.

4. R. Kent Nagle, Edward B. Saff, *Fundamentals of differential equations and boundary value problems*. Addison-Wesley Publishing Company, 1993.

Partial Differential Equations

Subject code: TTH306
Recommended Prerequisites: Real Analysis, Functional Analysis
Prerequisites: _
Student Workload: 170 hours, of which 50 hours of studying theories, 120 hours of independent study.
Number of Credits: 4

Contents: Study the existence of solutions of Elliptic, Hypebolic, Parabolic equations on Sobolev spaces.

Chapter 1: Elliptic equations
1.1. Sobolev spaces
1.2. Weak solutions, smoothness of solutions
1.3. Lax-Milgram theorem
1.4. Problems of eigenvalues and eigenvectors
1.5. The Maximum Principle

Chapter 2: Parabolic equations
2.1. Spaces $C(I, X)$
2.2. Theory of semigroups
2.3. Existence of solutions of parabolic equations
2.4. The Maximum Principle
2.5. Eigenvector expansions for the solution of Elliptic equations

Chapter 3: Hyperbolic equations
3.1. Existence of solutions of parabolic equations
3.2. Eigenvector expansions for the solution of Elliptic equations

Textbooks

1. Heim Brezis, *Functional Analysis, Sobolev spaces and partial differential equations. Springer*, 2011.
2. Paul Duchateau and David Zachman, Applied partial differential equations. Harper & Row, 1989.

References
1. Thierry Cazenave, Alain Haraux, *An introduction to semilinear evolution equations*. Clarendon Press, 1998.
2. Lawrence C. Evans, *Partial Differential Equations*. AMS, 2010.
3. Mikhailov, *Partial Differential Equations*. Mir Publisher, 1977.

Topology

Subject code: TTH309
Recommended Prerequisites: Analysis A2, Functional Analysis
Prerequisites: _
Student Workload: 170 hours, of which 50 hours of studying theories, 120 hours of independent study.
Number of Credits: 4

Contents: General Topology, I.e. point-set topology, including: topological spaces, continuity, homeomorphism, connectedness, convergence, compactness, product topology, Tikhonov theorem, Alexandroff compactification, Urysohn theorem, spaces of continuous functions, quotient topology.

Chapter 1: Infinite sets
1.1. Cardinality
1.2. Cardinalities of Z, Q, R, non-existence of maximal cardinality
1.3. The Axiom of choice

Chapter 2: Topological spaces
2.1. Topology
2.2. Interior, closure, boundary
2.3. Bases of a topology

Chapter 3: Subspaces
3.1. Subspace topology

Chapter 4: Connectedness
4.1. Connectedness, path-connectedness and relations
4.2. Connected components
4.3. Continuous functions on a connected space

Chapter 5: Separations
5.1. Hausdorff spaces, regular spaces, normal spaces, and their relations

Chapter 6: Convergence
6.1. Nets
6.2. Nets and convergence
6.3. Sequences and necessity of nets

Chapter 7: Compact spaces
7.1. Definition
7.2. Compactness in terms of sequences and relations
7.3. Lebesgue number of a compact metric space
7.4. Alexandroff compactification

Chapter 8: Product spaces
8.1. Product topology
8.2. Tikhonov theorem
8.3. Stone-Cech compactification

Chapter 9: Real functions and Spaces of functions
9.1. Urysohn lemma
9.2. The topology of uniform convergence and the compact-open topology
9.3. Tiestze extension theorem

Chapter 10: Quotient spaces
10.1. Quotient topology
10.2. Embedding and immersion
10.3. Some usual spaces: torus, Mobius surface, Klein surface, projective spaces

Textbooks
Huỳnh Quang Vũ, *Lecture notes on Topology*
http://www.math.hcmus.edu.vn/~hqvu/teaching/n.pdf

References
1. James R. Munkres, *Topology: A first course*, 2nd edition. Prentice-Hall, New Jersey, 2000.
2. Colin Adams, Robert Fransoza, *Introduction to Topology: Pure and Applied*. Pearson, 2009.

Measure Theory

Subject code: TTH322
Recommended Prerequisites: Analysis A1, Analysis A2, Analysis A3
Prerequisites: _
Student Workload: 170 hours, of which 50 hours of studying theories, 120 hours of independent study.
Number of Credits: 4

Contents:
Integrals related to positive measures: positive measures, measurable functions, integrable functions, Lebesgue's convergence theorems, Fatou's lemma.

Positive Borel measures: Riez's theorem (without proof). Regularity of positive Borel measures. Lebesgue's measure. Lusin theorem. Vitali–Carathéodory theorem. The space $L^1(R^n)$.

Integration on product spaces: measures on product spaces, Fubini's theorem.

Differentiation of measures: Derivatives of measures. Functions of bounded variation. Formula of change of variables.

References
1. J. Dieudonné, *Foundations of modern analysis*. Academic Press, New York, 1960.
2. J. T. Schwartz, *Nonlinear Functiomnal analysis and its applications*, Vol.I. Springer, New York, 1988.
3. W. Rudin, *Real and complex analysis*. McGraw-Hill, 1986.

Differential Geometry

Subject code: TTH340
Recommended Prerequisites: Analysis A1, Analysis A2, Analysis A3
Prerequisites: _
Student Workload: 170 hours, of which 50 hours of studying theories, 120 hours of independent study.
Number of Credits: 4

Contents: Differential geometry of curves and surfaces. Curvatures. Geodesics. Other topics.

Chapter 1: Curves
1.1. Curvatures of curves
1.2. Serret-Frenet theorem

Chapter 2: Surfaces
2.1. Surfaces - manifolds of dimension two
2.2. Regular surfaces and related results
2.3. Level sets corresponding to regular values

Chapter 3: Curvatures of surfaces
3.1. Gauss maps
3.2. Fundamental forms on surfaces
3.3. Gaussian curvatures, mean curvatures
3.4. Principal curvatures, normal curvatures
3.5. Gaussian curvatures in local coordinates
3.6. Isometries
3.7. Intrinsicity of Gaussian curvatures
3.8. Surfaces of constant curvatures, minimal surfaces, method of variation

Chapter 4: Geodesics
4.1. Geodesics, local equations
4.2. The exponential map, existence of geodesics
4.3. Gauss-Bonet theorem

Chapter 5: Other optional topics
Riemannian metrics on multi-dimensional manifolds, volumes, length of paths, distances
Hyperbolic geometry in dimensions 2 and 3, the disk model and the upper half space model

Textbooks
M. do Carmo, *Differential geometry of curves and surfaces*. Prentice-Hall, 1976.

References
1. John M. Lee, *Riemannian manifolds: an introduction to curvature*. Springer, 1997.

Nonlinear Operators

Subject code: TTH355

Recommended Prerequisites: Nonlinear Functional Analysis, Real Analysis, Functional Analysis, Calculus of Variation
Prerequisites: _
Student Workload: 170 hours, of which 50 hours of studying theories, 120 hours of independent study.
Number of Credits: 4

Contents:

Chapter 1: Basic operators
1.1. Nemytskii operator
1.2. Uryson operator
1.3. Nonlinear integral operator

Chapter 2: Compact vector fields
2.1. Topological degrees of compact vector fields
2.2. Differentiable compact vector fields
2.3. Index of a fixed point

Chapter 3: Operators of class $S_{+¿¿}$
3.1. Definition
3.2. Topological degrees for operators of class $S_{+¿¿}$

Chapter 4: Application to partial differential equations
4.1. Weak solutions of partial differential equations
4.2. Non-trivial solutions of partial differential equations

References
1. D. M. Duc, *Giải tích hàm*. NXB ĐHQG TPHCM, 2000.
2. M. A. Krasnosel'skii, *Topological methods in the theory of nonlinear integral equations*. Pergamon Press, Oxford, 1964.
3. I. V. Skrypnik, *Methods for analysis of nonlinear elliptic boundary value problems*. AMS, Providence, 1994.

Calculus of Variation

Subject code: TTH356
Recommended Prerequisites: Analysis A1, Analysis A2, Analysis A3, Functional Analysis, Nonlinear Functional Analysis

Prerequisites: _
Student Workload: 170 hours, of which 50 hours of studying theories, 120 hours of independent study.
Number of Credits: 4

Contents:

Chapter 1: Differentiation in normed spaces
1.1. Differentiability
1.2. Inverse function theorem and applications
1.3. Lagrange multipliers

Chapter 2: Lower semi-continuity
2.1. Definition
2.2. The greatest lower bound of a lower semi-continuous function

Chapter 3: Mountain-pass theorem
3.1. Theorem
3.2. Topological degrees at limit points

Chapter 4: Applications to partial differential equations
4.1. Weak solutions of partial differential equations
4.2. Non-trivial solutions of partial differential equations

References
1. D. M. Duc, *Giải tích hàm*. NXB ĐHQG TPHCM, 2000.
2. Michael Struwe, *Variational Methods, Applications to Nonlinear Partial Differential Equations and Hamiltonian Systems*. Sringer, 2008.

Algebraic Topology

Subject code: TTH357
Recommended Prerequisites: Topology
Prerequisites: _
Student Workload: 170 hours, of which 50 hours of studying theories, 120 hours of independent study.
Number of Credits: 4

Contents: Simplicial and cellular complexes, homotopy, fundamental groups, simplicial homology, singular homology, cellular homology.

Chapter 1: Simplicial and cellular complexes
1.1. Simplicial complexes, polyhedrons, triangulation, cutting and pasting; investigating some important examples
1.2. Classification of compact surfaces
1.3. Euler characteristics
1.4. Cell complexes; important examples: Lens spaces, projective spaces, Seifert spaces, Poincaré dodecahedron space,...

Chapter 2: Homotopy
2.1. Fundamental groups
2.2. Fundamental group of circle
2.3. Seifert-Van Kampen theorem
2.4. Fundamental groups of cell complexes; examples

Chapter 3: Homology
3.1. Simplicial homology
3.2. Singular homology
3.3. Mayer-Vietoris theorem, homology groups of sphere
3.4. Cellular homology; calculation and examples
3.5. Relationship between fundamental groups and homology groups, Betti numbers, Euler characteristics

Chapter 4: Applications (further reading)
Jordan theorem, Invariance of domain, Brouwer fixed-point theorem, Borsuk-Ulam theorem; applications to computational topology...

Textbooks
Huỳnh Quang Vũ, *Lecture notes on Topology*
http://www.math.hcmus.edu.vn/~hqvu/teaching/n.pdf

References
1. Seifert H., Threlfall W., *A textbook of Topology*. Academic Press, 1980.
2. Vassiliev V. A., *Introduction to Topology*. AMS, 2001.
3. Vick James, *Homology Theory*. Springer, 1994.
4. Hatcher Allen, *Algebraic Topology*. Cambridge, 2001.

Modern Algebra

Subject code: TTH501
Recommended Prerequisites: _
Prerequisites: Higher Algebra
Student Workload: 170 hours, of which 50 hours of studying theories, 120 hours of independent study.
Number of Credits: 4

Contents:

Chapter 1: Group theory
1.1. Groups and group homomorphisms
1.2. Lagrange theorem
1.3. Isomorphism theorems
1.4. Actions of a group on a set
1.5. Conjugation actions
1.6. Finite p-groups
1.7. Sylow theorems
1.8. Solvable groups
1.9. Nilpotent groups
1.10. Frattini subgroups
1.11. Hall subgroups and Carter subgroups
1.12. Free Abelian groups
1.13. Free groups
1.14. Finitely generated Abelian groups

Chapter 2: Rings, Integral domains and Fields
2.1. Rings
2.2. Isomorphism theorem
2.3. Polynomial rings
2.4. Principal ideal domains and Euclidean domains
2.5. Unique factorization domains
2.6. Dedekind domains

References:
1. William A. Adkins, Steven H. Weintraub, *Algebra: An appproach via Module theory*. Springer-Verlag, New York, 1992.
2. Daniel Gorenstein, *Finite groups*. Harper & Row, 1968.
3. M. I. Kargapolov, Ju. I. Merzljakov, *Fundamentals of the Theory of Groups*. Springer-Verlag, 1979.
4. A. I. Kostrikin, I. R. Shafarevich, *Algebra IV*. EMS 37, Springer-Verlag, 1995.

5. Joseph J. Rotman, *An Introduction to the Theory of Groups*. Springer-Verlag, 1995.

Field and Galois Theory

Subject code: TTH502
Recommended Prerequisites: _
Prerequisites: Higher Algebra
Student Workload: 170 hours, of which 50 hours of studying theories, 120 hours of independent study.
Number of Credits: 4

Contents:

Chapter 1: Preliminaries
1.1. Permutation groups
1.2. Solvable groups
1.3. Polynomials

Chapter 2: Galois theory
2.1. Finite extensions and algebraic extensions
2.2. Galois groups and fixed subfields
2.3. Splitting fields and normal extensions
2.4. Separable extensions
2.5. Normal closures
2.6. Purely inseparable extensions
2.7. The Fundamental Theorem of Galois theory
2.8. Some examples

Chapter 3: Applications of Galois theory
3.1. The Fundamental Theorem of Algebra
3.2. Polynomials of small degrees
3.3. Transcendental numbers π and e
3.4. Ruler-and-compass constructions
3.5. Solvability by Radicals
3.6. An insolvable quintic

Chapter 4: Some Galois extensions (optional)
4.1. Finite fields
4.2. Cyclotomic extensions

4.3. Traces and norms
4.4. Cylic extensions
4.5. The Hilbert 90 theorem and Cohomology of groups
4.6. Kummer extensions

References

1. Ian Stewart, *Galois theory*. Chapman and Hall, 1973.
2. Patrick Morandi, *Field and Galois theory*. GTM 167, Springer, 1996.

Ring Theory

Subject code: TTH503
Recommended Prerequisites: _
Prerequisites: Higher Algebra
Student Workload: 170 hours, of which 50 hours of studying theories, 120 hours of independent study.
Number of Credits: 4

Contents:

Chapter 0: Preliminaries
0.1. Maps
0.2. Ordering relations
0.3. The axiom of choice
0.4. Cardinalities

Chapter 1: Rings
1.1. Definition and examples
1.2. Ideals and homomorphisms; quotient rings
1.3. Ring isomorphism theorems
1.4. Direct products and direct sums of rings
1.5. Direct subproducts of rings
1.6. Prime ideals and maximal ideals
1.7. Radical properties of rings
1.8. Prime rings and semiprime rings
1.9. Commutative local rings
1.10. Commutative semilocal rings
1.11. Rings of fractions

Chapter 2: Modules
2.1. Definition and basic properties
2.2. Submodules and linear combinations
2.3. Quotient modules and homomorphisms
2.4. Annihilators
2.5. Direct products and direct sums of modules
2.6. Direct subproducts of modules
2.7. Chain conditions on modules
2.8. Free modules and projective modules

Chapter 3: Tensor products and Algebras (optional)

References
1. I. N. Herstein, *Noncommutative Rings*. The Carus Mathematical monographs, 1968.
2. I. N. Herstein, *Topics in Algebra*. Xeros College Publishing, Toronto, 1964.
3. P. K. Draxl, *Skew Fields*. Lecture note series 81, London Mathematical society, 1993.
4. T. Y. Lam, *The First Course in Noncommutative Rings*. GTM Vol.13, Springer-Verlag, 1996.
5. Frank W. Anderson, Kent R. Fuller, *Ring and Categories of modules*. GTM, Springer-Verlag, 1973.
6. Charles W. Cutis, *Representation Theory of finite Groups and associative Algebras*. Interscience publishers, New York-London, 1962.
7. Berson Frad and R. Keith Dennis, *Noncommutative Algebra*. GTM 144, Springer-Verlag, 1991.
8. M. F. Atiyah, L. G. Macdonald, *Introduction to commutative Algebra*. Addison-Wesley Publishing Company, Springer-Verlag, 1969.

Graduation Thesis

Subject code: TTH357
Recommended Prerequisites: _
Prerequisites: _
Student Workload: 675 to 800 hours, of which 300 hours of independent study.
Number of Credits: 10