

## Preface: Phan Quoc Khanh, Special Issue on Optimization and Variational Analysis Honoring His 65th Birthday

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Professor Phan Quoc Khanh

This issue of the Vietnam Journal of Mathematics, entitled “*Optimization and Variational Analysis*”, is dedicated to Professor Phan Quoc Khanh on the happy occasion of his 65th birthday. It consists of fifteen invited papers covering several areas of Optimization and Variational Analysis; the authors have close academic ties to Professor Khanh.

Optimization theory and its various applications are highly important in the developing of advanced technologies, economics, management, and other aspects of the modern society. From the mathematical viewpoint, optimization theory and numerical algorithms belong to the challenging areas of applied mathematics and require the development of new form of analysis.

Variational analysis offers new ideas and techniques, which from one hand are largely motivated by applications to optimization, while from the other hand employ variational principles and optimization methodology to analyze and solve numerous problems that may not have any variational structures.

Since nondifferentiable functions, sets with nonsmooth boundaries, and set-valued mappings naturally and frequently appear in the development and applications of modern variational principles and techniques, methods of generalized differentiation play a prominent role in variational analysis.

Vietnamese mathematicians have been well recognized among the best international experts in the areas of optimization, variational analysis, and generalized differentiation. Professor Khanh belongs to the distinguished group of Vietnamese and international leaders in various aspects of optimization and variational analysis, and that is why this special issue is dedicated to him. There does not seem to be better place than this to write a short biography on him, highlight his various milestones and achievements.

Professor Khanh is one of the leading mathematicians of Vietnam. He has published around 100 papers in international mathematical journals, which have received an extensive record of citations. His works have contributed significantly to the development of the fields of optimization and variational analysis. He has been invited for scientific stays at a number of Mathematics institutions. He has supervised 14 doctoral (10 defended) students and more than 100 master's students. The big and very active Optimization group of southern Vietnam, founded and led by Khanh, is well-known in the international optimization community and consists mainly of generations of his students who are now playing crucial roles in various scientific and educational institutions in Ho Chi Minh City and southern provinces.

The scientific community of Vietnam respects Professor Khanh not only for his excellent scientific career, but also for his passion and enthusiasm for Mathematics. Khanh was born on the 2nd of September, 1946, in Hanoi, Vietnam. In 1964 Khanh enrolled in the Hanoi University which was evacuated to Dai Tu, a remote and mountainous district of Thai Nguyen province because of the extension of the US military air attacks into the northern areas of the country. They had to teach and study under extremely difficult conditions. They had to build temporary cottages, to make tables and chairs as well as other necessities. The classes were held in underground shelters in the jungle. They studied without textbooks and without gasoline for light. They, usually three or four, shared an oil lamp to read their notes. Overcoming all the numerous difficulties, Khanh managed to graduate with outstanding honor and was enlisted in the military service at the request of his country. He joined the Vietnam Military Academy of Technology in 1968. In many difficult years thereafter he has been known for his relentless efforts in conducting research and for his beautiful results, which were achieved despite a severe shortage of books, journals, references, and other resources.

Khanh's Ph.D. dissertation, in the field of Optimal Control, was defended in 1978 at the Institute of Mathematics, Polish Academy of Sciences, Warsaw,

under the supervision of Professor Stefan Rolewicz. Nine years later, at the same institute, he successfully completed his Doctor of Science degree (Habilitation). In 1993, he left the army at the rank of colonel, moved to Ho Chi Minh City, and began a new and productive phase of his professional career. Later that year, he became the Chair of the Department of Mathematics and Computer Science at the University of Ho Chi Minh City. Soon after, he founded the new Section of Optimization and System Theory within this department, offering both undergraduate and graduate programs. He also held several important positions in other universities.

The main focus of Khanh's research has been on variational and set-valued analysis and multiobjective optimization. Khanh has discovered very deep and powerful results on open mappings and implicit functions in his Ph.D. dissertation and later on, which have led him to new developments in the areas of multiobjective optimization, equilibrium problems, and variational inequalities. His concept of variational sets and their applications to multiobjective optimization are among the major achievements of variational analysis that allow us to handle optimization problems of any structure in general space settings. His recent results on stability of variational systems and variational relations have also received a strong appreciation to experts of the field.

Khanh has made crucial contributions to the development of mathematical education in the south of Vietnam. He revised and proposed several mathematical curricula and training programs and founded a number of units in several universities and institutes. His seminars attract regular participation of great many young mathematicians, who travel long distances from the provinces of Can Tho, Da Lat, Nha Trang, Buon Me Thuot, Dong Thap, etc. to attend Khanh's seminars. These seminars are the core of the activities of the Optimization Group of Southern Vietnam and are visited also by many foreign professors.

Khanh was among the founding members of the International University of Ho Chi Minh City, a new model for universities of excellence in Vietnam, and served as its first rector. He also founded the Department of Mathematics of this university and has been its chair until now. In addition, Khanh is an active member of many important scientific committees and organizations at the national and international level. For instance, he is a Vice President of the Vietnam Mathematical Society, Vice Chair of the Mathematics Council of the National Foundation for Science and Technology Development, a member of the National Executive Board for the Development of Mathematics, and a member of the National Committee for Professorship in Mathematics.

We are glad that the Vietnam Journal of Mathematics agreed to publish this special issue celebrating Professor Khanh's 65th birthday. We are very grateful to all of the contributing authors as well as to the many reviewers for their invaluable and indispensable support.

At this occasion, we would like to express our deepest respect to Professor Phan Quoc Khanh and to wish him good health and many productive years to come.

**Selected Papers of Professor Phan Quoc Khanh**

1. Phan Quoc Khanh, An induction theorem and general open mapping theorems, *J. Math. Anal. Appl.* **118** (1986), 519–534.
2. Phan Quoc Khanh, An open mapping theorem for families of multifunctions, *J. Math. Anal. Appl.* **132** (1988), 491–498.
3. Phan Quoc Khanh, On general open mapping theorems, *J. Math. Anal. Appl.* **144** (1989), 305–312.
4. Phan Quoc Khanh, On Caristi-Kirk's theorem and Ekeland's variational principle for Pareto extrema, *Bull. Polish Acad. Sci. Math.* **37** (1989), 33–39.
5. Phan Quoc Khanh, Optimality conditions via norm scalarization in vector optimization, *SIAM J. Control Optim.* **31** (1993), 646–658.
6. Phan Quoc Khanh and Nguyen Dinh Tuan, Variational sets of multivalued mappings and a unified study of optimality conditions, *J. Optim. Theory Appl.* **139** (2008), 47–65.
7. Nguyen Le Hoang Anh, Phan Quoc Khanh and Le Thanh Tung, Variational sets: calculus and applications to nonsmooth vector optimization, *Nonlinear Anal.* **74** (2011), 2358–2379.